PERGAMON

International Journal of Solids and Structures 38 (2001) 7101–7111

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

# An analysis of pressure–frequency characteristics of vibrating string-type pressure sensors

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Received 27 July 2000; in revised form 10 May 2001

**Abstract**

This article presents a numerical analysis on the pressure–natural frequency characteristic of a pre-tensioned string-plate pressure sensor. The sensor is made of a thin circular plate and a pre-tensioned string attached at the center of the plate. When the plate is subjected to a pressure, the tensile force in the string changes. Thus, the string's natural frequency varies with the pressure on the plate. The von Karman plate theory incorporating geometric nonlinearities is employed to study the interaction between deflection of the plate and tension of the string. Using numerical method based on series solution to analyze the nonlinear bending of the plate, a natural frequency–pressure characteristic to the sensor is formulated. Finally, the method is implemented to predict the characteristic that is useful for design of sensors. © 2001 Elsevier Science Ltd. All rights reserved.

*Keywords:* Pressure sensor; Vibrating string; Thin plate; Geometrically nonlinear deflection; Pressure–natural frequency characteristic; Theoretical analysis

## 1. Introduction

The research on design of pressure sensors has played an important role in engineering applications. Until now, various pressure sensors have been designed to be extensively used in practice. In the last 20 years, a new kind of pressure sensor has been proposed; it measures the frequency of an elemental structure of the sensor and computes pressure through a pressure–frequency characteristic. One of the sensors is made of a thin circular diaphragm/plate and a string which is attached at the center of the diaphragm as shown in Fig. 1. When a pressure is applied on the diaphragm, deformation of the diaphragm causes variation of tensile force of the string, which leads to change of frequency of the string. Through measuring the natural frequency of the string by arrangement of electromagnetically stimulated vibration and vibration pickup, one obtains the magnitude of the external pressure by means of its pressure–frequency characteristic (Liu and Zhuang, 1981). This kind of pressure sensor has become the core element of digital-type atmosphere output devices.

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PII: S0020-7683(01)00154-8

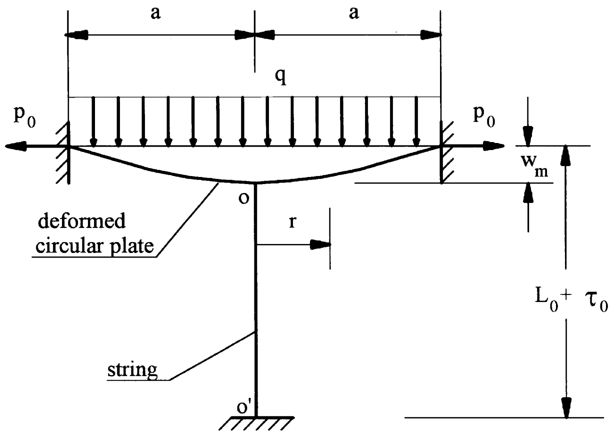


Fig. 1. Schematic drawing of the vibrating-string type pressure sensor.

According to Liu and Zhuang (1981) and Zhou et al. (1994), we know that the research on the vibrating diaphragm-string type pressure sensors is rather inadequate, e.g., the effect of geometric nonlinearity of deflection of plate of the sensor is not considered in theoretical analysis of the pressure–frequency characteristic. But we know, the diaphragm is so thin that its deflection is sensitive to the nonlinearity (Zhou et al., 1994). Usually, the von Karman equations of thin plates are used to describe the geometrical deformation of thin plates under a normal load (von Karman, 1910). And there are various well established methods to solve these nonlinearly coupled equations, such as perturbation method, iterative method, and series expansion, etc. (e.g. Friedrichs and Stoker, 1941; Chien, 1947; Keller and Reiss, 1958; Schmidt, 1968; Schmidt and Dadeppo, 1975; Yeh et al., 1989; Zheng and Zhou, 1986, 1990; Zhou and Tzou, 2000). Some studies of convergence for them were conducted by Keller and Reiss (1958), Zheng and Zhou (1986, 1990), and Zheng and Lee (1995), etc. In this paper, the interaction between the deformations of diaphragm/plate and string is taken into account in the theoretical analysis. In this case, the diaphragm/plate is subjected to a uniform pressure and a concentrated load caused by the tensile force exerted at the center of the plate. In order to produce accurate numerical results of solution of the problem, a series solution is chosen to solve the nonlinear differential equations of the loaded plate, which reduces the differential equations to a set of algebraic equations. After that, an embedded Raphson–Newtonian method of iteration is employed to solve the nonlinear algebraic equations, from which the solutions of the problem are obtained, and a formula for the natural frequency of the string is expressed by the interaction between the plate and the string. Finally, the semi-analytical and semi-numerical program is carried out for a case study. And the pressure–natural frequency characteristics of the pressure sensor are displayed for different design parameters.

## 2. Fundamental equations

Consider the pressure sensor made of a thin circular plate with radius $a$ and thickness $h$ and a string with original length $L_0$. Denote the Young modulus of the plate by $E_p$ and the Young modulus of the string by $E_s$. Let $\tau_0$ be the length of the gap spand by the string at one end, denoted by $o$, to the center of undeformed plate. The other end of the string, denoted by $o'$, is where the string is rigidly anchored (see Fig. 1). After the $o$ end of the string is connected with the central point of the elastic element or circular plate, the string will be elongated and the plate be bent. With application of uniform pressure $q$ to outer surface of the plate, the internal tensile force $T$ in the string changes, further, its natural frequency varies. In order to get this relation between the pressure and the frequency, we make some simplifying assumption: (1) the free vibration in the vicinity of static deformation is so small that its effect on static deformation of the sensor is negligible; (2) the plate and the string are homogeneous; (3) the deflection of the plate is geometrically nonlinear, following von Karman's theory of plates. For this case, the plate is subjected to a distributed pressure $q$ and a concentrated load $T$ at its center. It is obvious that the tensile force is coupled to deflection of the plate.

### 2.1. Elemental equation of vibrating string

From the theory of small vibrations of elongated string, one can write the natural frequencies of the string with tension force $T$ in the form

$$\omega_n = n\pi\sqrt{\frac{T}{\rho L_0^2}} \quad n = 1, 2, 3, \ldots \tag{1}$$

in which $\rho$ is the mass per unit length of the string. Usually, the fundamental natural frequency, corresponding to $n = 1$, is most useful in practice.

Denote the elongation of the string by $\Delta L$. From the mechanics of materials, then, we have

$$\Delta L = \frac{TL_0}{E_s F} \tag{2}$$

in which $F$ is the cross-sectional area of the string. Let

$$k = \frac{E_s F}{L_0} \tag{3}$$

One can write

$$T = k\Delta L \tag{4}$$

### 2.2. Nonlinear bending of plate

For the circular plate considered here, when the geometrically nonlinear deformation of the von Karman type is taken into account, one can write its governing equation of the nonlinear bending of axisymmetric deflection of the form (Zhou and Zheng, 1989):

$$D\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}w}{\mathrm{d}r}\right)\right] = N_r\frac{\mathrm{d}w}{\mathrm{d}r} + \frac{qr}{2} + \frac{T}{2\pi r} \tag{5}$$

$$r\frac{\mathrm{d}}{\mathrm{d}r}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r^2 N_r) = -\frac{E_p h}{2}\left(\frac{\mathrm{d}w}{\mathrm{d}r}\right)^2 \quad 0 < r < 0 \tag{6}$$

and the boundary conditions:

$$r = 0: \quad \frac{\mathrm{d}w}{\mathrm{d}r} = 0, \quad N_r \text{ finite} \tag{7}$$

$$r = a: \quad w = \frac{\mathrm{d}w}{\mathrm{d}r} = 0, \quad \frac{u_0}{a} = \frac{1}{E_p h}\left[a\frac{\mathrm{d}N_r}{\mathrm{d}r} + (1 - v)N_r\right] \tag{8}$$

Here $r$ is the radial distance from the center of the plate; $w$ is the transverse deflection of the plate; $N_r$ denotes the radial membrane force per unit length of the plate; $v$ is the Poisson's ratio of the plate; $h$ is the thickness of the plate; and $D = E_p h^3/[12(1-\nu^2)]$ is the flexural rigidity of the plate. Here, the outer boundary edge at $r = a$ is rigidly clamped with a pre-given radial displacement $u_0$ which is generated by an initial membrane tensile force $p_0$ along radial direction of the plate at the edge $r = a$, i.e., they have a relation

$$u_0 = \frac{(1-\nu)a}{Eh} p_0 \tag{9}$$

### 2.3. Connection condition

For the pressure sensor with the vibrating plate-string structure in Fig. 1, the following geometrical relation between the elongation $\Delta L$ of the string and the maximum deflection $w_m = w(r)|_{r=0}$ of the plate holds:

$$\tau_0 = w_m + \Delta L \tag{10}$$

Obviously, if the case $w_m = \tau_0$ occurs when the plate is subjected to a pressure $q^*$, we have $\Delta L = 0$ and $T = 0$, further, $\omega_n = 0$. If the pressure increases over $q^*$, thus, the pressure sensor fails to measure the pressure. In other words, the pressure sensor is efficient only when $q < q^*$.

Substituting Eq. (4) into Eq. (10), we get

$$\tau_0 = w_m + \frac{T}{k} \tag{11}$$

which is clearly coupled to the nonlinear bending Eqs. (5)–(8). By solving the nonlinear boundary-value problem of Eqs. (5)–(8) under condition of Eq. (11), we can get the tensile force $T$ which varies with applied pressure $q$ and initial force $p_0$, which is given in the next section by means of a semi-analytical solution of series expansion. After that, the pressure–natural frequency characteristic of the sensor may be derived from Eq. (1).

## 3. Series solution to nonlinear boundary-value problem

For the simplicity of discussion, here, we introduce the following dimensionless quantities:

$$\begin{gathered} y = \left(\frac{r}{a}\right)^2, \quad W = \sqrt{3(1-\nu^2)}\frac{w}{h}, \quad \phi(y) = y\frac{\mathrm{d}W}{\mathrm{d}y}, \quad \tau = \sqrt{3(1-\nu^2)}\frac{\tau_0}{h} \\ S(y) = \frac{3(1-\nu^2)a^2 N_r}{E_p h^3} y, \quad K = \frac{3(1-\nu^2)a^2}{4\pi E_p h^3} k, \quad U_0 = \frac{3(1+\nu)a}{h^2} u_0 \\ Q = [3(1-\nu^2)]^{3/2} \frac{a^4 q}{4 E_p h^4}, \quad P = [3(1-\nu^2)]^{3/2} \frac{a^2 T}{4\pi E_p h^4}, \quad S_0 = \frac{3(1-\nu^2)a^2}{E_p h^3} p_0 \end{gathered} \tag{12}$$

Then, the Eqs. (5)–(8) and (11) can be reduced into the nondimensionlized form (Zheng and Zhou, 1989):

$$y^2 \frac{\mathrm{d}^2\phi(y)}{\mathrm{d}y^2} = \phi(y)[S^*(y) + S_0 y] + Qy^2 + Py \tag{13}$$

$$y^2 \frac{\mathrm{d}^2 S^*(y)}{\mathrm{d}y^2} = -\frac{1}{2}\phi^2(y) \quad 0 < y < 1 \tag{14}$$

$$y = 0: \quad \phi(y) = 0, \quad S^*(y) = 0 \tag{15}$$

$$y = 1: \quad \phi(y) = \frac{\lambda}{\lambda - 1}\frac{d\phi(y)}{dy}, \quad S^*(y) = \frac{\mu}{\mu - 1}\frac{dS^*(y)}{dy} \tag{16}$$

$$\tau = W_m + P/K \tag{17}$$

where

$$W_m = -\int_0^1 \frac{1}{\eta}\phi(\eta)d\eta \tag{18}$$

is the central deflection of the plate with dimensionless form, and

$$S^*(y) = S(y) - S_0 y \tag{19a}$$

$$S_0 = U_0 \tag{19b}$$

$$\lambda = \frac{2}{1 + v + k_1 a/D} \tag{19c}$$

$$\mu = \frac{2}{1 - v + Eh/k_2} \tag{19d}$$

Here, $k_1$ and $k_2$ are respectively rigidity coefficients of the radial rotation and radial displacement at outer edgy of the plate. For the case of clamped edgy at $y = 1$, we have $k_1 \to \infty$ and $k_2 \to \infty$ thus $\lambda = 0$ and $\mu = 2/(1 - v)$.

To solve the nonlinear differential equations (13) and (14) with conditions of Eqs. (15)–(17), here, we employ the series expansion (Schmidt, 1968; Zheng and Zhou, 1990).

$$\phi(y) = \sum_{i=1}^{\infty}\sum_{j=0}^{i} A_{ij} y^i \ln^j y \tag{20}$$

$$S^*(y) = \sum_{i=1}^{\infty}\sum_{j=0}^{i} B_{ij} y^i \ln^j y \tag{21}$$

Zheng and Zhou (1990) gave a study of convergence of the series solutions to boundary-value problem of the von Karman's theory of circular plates under a uniform distributed pressure and a concentrated center load together. Substituting of the series expansions of Eqs. (20) and (21) into the differential equations (13) and (14), and comparing coefficients of the terms $y^i \ln^j y$ $(i = 1, 2, \ldots;\ j = 0, 1, 2, \ldots, i)$, we obtain the following recurrent formulas for determining the unknown constants $A_{ij}$ and $B_{ij}$:

$$\begin{aligned} &A_{11} = P, \quad B_{11} = 0, \quad A_{22} = 0, \quad A_{21} = \tfrac{1}{2}A_{11}B_{10} + \tfrac{1}{2}S_0A_{11}, \quad A_{20} = \tfrac{1}{2}A_{10}B_{10} + \tfrac{1}{2}Q - \tfrac{3}{2}A_{21} + \tfrac{1}{2}S_0A_{10}, \\ &B_{22} = -\tfrac{1}{4}A_{11}^2, \quad B_{21} = -\tfrac{1}{4}A_{11}A_{10} - 3B_{22}, \quad B_{20} = -\tfrac{1}{4}A_{10}^2 - \tfrac{3}{2}B_{21} - B_{22} \end{aligned} \tag{22}$$

and

$$A_{ij} = \frac{1}{i(i-1)}\left[\sum_{l=1}^{i-1}\sum_{k=0}^{j} A_{i-l,j-k}B_{lk} - (j+1)(2i-1)A_{i,j+1} - (j+1)(j+2)A_{i,j+2} + S_0A_{i-1,j}\right] \tag{23a}$$

$$B_{ij} = \frac{1}{i(i-1)}\left[-\frac{1}{2}\sum_{l=1}^{i-1}\sum_{k=0}^{j}A_{i-l,j-k}A_{lk} - (j+1)(2i-1)B_{i,j+1} - (j+1)(j+2)B_{i,j+2}\right]$$
$$i = 3,4,5,\ldots;\quad j = i, i-1,\ldots,2,1,0 \tag{23b}$$

It is obvious from the above recurrent formulas that only constants $A_{10}$, $B_{10}$, and the reaction $P$ are unknowns. That is, once they are known, we can get the values of all constants $A_{ij}$ and $B_{ij}$ in the Eqs. (20) and (21) such that the solutions of the problem are gained. It is noted that the expression of series solutions of Eqs. (20) and (21) satisfies the boundary conditions of Eq. (15). Substituting the series solutions to the boundary conditions of Eq. (16) and the constraint of Eq. (11) through Eq. (18), we get

$$\sum_{i=1}^{\infty}\{[\lambda(1-i)-1]A_{i0} - \lambda A_{i1}\} = 0 \tag{24}$$

$$\sum_{i=1}^{\infty}\{[\mu(1-i)-1]B_{i0} - \mu B_{i1}\} = 0 \tag{25}$$

and

$$\left\{\sum_{i=1}^{\infty}\sum_{j=0}^{i}\frac{(-1)^{j+1}\cdot j!}{i^{j+1}}A_{ij} - \tau\right\}K + P = 0 \tag{26}$$

which are nonlinear algebraic equations for determining the unknowns $A_{10}$, $B_{10}$ and $P$ through Eqs. (22), (23a) and (23b). By solving the nonlinear algebraic equations, we may get the solutions of nonlinear bending of plate and the reaction $P$ or $T$ of the pressure sensor as long as that the parameters $\tau$, $S_0$, $K$, and $Q$ are given. For a practical sensor, the first three parameters are usually pre-specified. In this case, the reaction $P$ varies with pressure $q$ or $Q$ which is subjected to the plate.

## 4. Analytical pressure–frequency characteristic

Before we give a solving method to the solutions introduced in the previous section, we give an analytical formula of pressure–natural frequency characteristic once the unknown $P$ is known, which is useful to the practical design of the pressure sensor. According to the expression of dimensionless quantity $P$, we have

$$T = \frac{E_{\mathrm{p}}h^4}{a^2b_1}P \tag{27}$$

where

$$b_1 = \frac{3}{4\pi}(1-v^2)\sqrt{3(1-v^2)} \tag{28}$$

Substitution of Eq. (27) into Eq. (1) leads to

$$\omega_n = n\omega^*\frac{h^2}{aL_0}\sqrt{\frac{E_{\mathrm{p}}}{\rho}} \tag{29a}$$

in which

$$\omega^* = \pi\sqrt{\frac{P}{b_1}} = \pi\sqrt{\frac{4\pi P}{[3(1-v^2)]^{3/2}}} \tag{29b}$$

Eq. (29a) tells us that the fundamental natural frequency of the string in the pressure sensor varies with proportion of square of thickness of the plate, $h^2$, and square root of the Young modulus, $\sqrt{E_p}$, and with inverse ratio of radius of the plate, $a$, the length of the string, $L_0$, and square root of mass density of the string, $\sqrt{\rho}$. Of course, it is certain that the reaction $T$ or $P$ is related to these parameters too through $Q$, $S_0$, $K$, $\tau$. Due to the dimensionless form of quantities, the results given here may be more extensively applied in practice once the relation $\omega^* = \omega^*(Q)$ is obtained by means of Eq. (29b) and the solution of nonlinear algebraic equations (24)–(26).

## 5. Semi-numerical program and results (case study)

Here, we employ the Raphson–Newtonian method to solve the nonlinear algebraic Eqs. (24)–(26) for numerical results of the problem considered. In order to enhance the convergence of the method, we choose an embedded iteration with some physical meaning. Denote

$$X = \sum_{i=1}^{\infty}\{[\lambda(1-i)-1]A_{i0} - \lambda A_{i1}\} \tag{30}$$

$$Y = \sum_{i=1}^{\infty}\{[\mu(1-i)-1]B_{i0} - \mu B_{i1}\} \tag{31}$$

$$Z = \left\{\sum_{i=1}^{\infty}\sum_{j=0}^{i}\frac{(-1)^{j+1}\cdot j!}{i^{j+1}}A_{ij} - \tau\right\}K + P \tag{32}$$

*Step 1*: we solve the root of equations

$$X = 0 \quad \text{and} \quad Y = 0$$

for a pre-specific $P = P^*$ and a given pressure $Q$ or $q$ by means of the Raphson–Newtonian method. That is,

$$\bar{A}_{10} = A_{10}^* + \xi_1 \tag{33}$$

$$\bar{B}_{10} = B_{10}^* + \xi_2 \tag{34}$$

in which $A_{10}^*$ and $B_{10}^*$ are iterating values of $A_{10}$ and $B_{10}$, respectively; $\bar{A}_{10}$ and $\bar{B}_{10}$ are the iterated values of them; $\xi_1$ and $\xi_2$ are the revised values which are determined by

$$\xi_1 = \left\{Y\frac{\partial X}{\partial B_{10}} - X\frac{\partial Y}{\partial B_{10}}\right\} \Big/ \left\{\frac{\partial X}{\partial A_{10}}\frac{\partial Y}{\partial B_{10}} - \frac{\partial X}{\partial B_{10}}\frac{\partial Y}{\partial A_{10}}\right\}\Bigg|_{\left(A_{10}^*,B_{10}^*\right)} \tag{35}$$

$$\xi_2 = \left\{X\frac{\partial Y}{\partial A_{10}} - Y\frac{\partial X}{\partial A_{10}}\right\} \Big/ \left\{\frac{\partial X}{\partial A_{10}}\frac{\partial Y}{\partial B_{10}} - \frac{\partial X}{\partial B_{10}}\frac{\partial Y}{\partial A_{10}}\right\}\Bigg|_{\left(A_{10}^*,B_{10}^*\right)} \tag{36}$$

where

$$\frac{\partial X}{\partial A_{10}} = \sum_{i=1}^{\infty}\left\{[\lambda(1-i)-1]\frac{\partial A_{i0}}{\partial A_{10}} - \lambda\frac{\partial A_{i1}}{\partial A_{10}}\right\} \tag{37}$$

$$\frac{\partial X}{\partial B_{10}} = \sum_{i=1}^{\infty}\left\{[\lambda(1-i)-1]\frac{\partial A_{i0}}{\partial B_{10}} - \lambda\frac{\partial A_{i1}}{\partial B_{10}}\right\} \tag{38}$$

$$\frac{\partial Y}{\partial A_{10}} = \sum_{i=1}^{\infty} \left\{ \mu(1-i) - 1] \frac{\partial B_{i0}}{\partial A_{10}} - \mu \frac{\partial B_{i1}}{\partial A_{10}} \right\} \tag{39}$$

$$\frac{\partial Y}{\partial B_{10}} = \sum_{i=1}^{\infty} \left\{ [\mu(1-i) - 1] \frac{\partial B_{i0}}{\partial B_{10}} - \mu \frac{\partial B_{i1}}{\partial B_{10}} \right\} \tag{40}$$

Similarly, taking the partial differentiation of Eqs. (22) and (23) with respect to $A_{10}$ and $B_{10}$ respectively, we can get a set of recurrent formulas of the partial differentiation $\partial A_{ij}/\partial A_{10}$, $\partial A_{ij}/\partial B_{10}$, $\partial B_{ij}/\partial A_{10}$, and $\partial B_{ij}/\partial B_{10}$ only when we choose

$$\frac{\partial A_{10}}{\partial A_{10}} = 1, \quad \frac{\partial A_{10}}{\partial B_{10}} = 0$$

$$\frac{\partial B_{10}}{\partial A_{10}} = 0, \quad \frac{\partial B_{10}}{\partial B_{10}} = 1$$

Replacing the iterating values of $A_{10}$ and $B_{10}$ by their iterated ones in each iterative step, and repeating the above calculations until the precision conditions

$$|X| < 10^{-5}, \quad |Y| < 10^{-5} \tag{41}$$

are fulfilled, we approximately get the root of Eqs. (24) and (25), denoted by $A_{10}^*$ and $B_{10}^*$. It is evident that these roots are dependent upon the pre-specific value $P = P^*$. That is, we have $A_{10}^* = A_{10}^*(P^*)$, and $B_{10}^* = B_{10}^*(P^*)$.

*Step 2*: In order to satisfy Eq. (26), or $Z = 0$, we choose $P^*$ to be an iterating value of $P$ for solving the nonlinear algebraic equation $Z = 0$ with unknown $P$. Applying the Raphson–Newtonian method of order 1 to Eq. (26), we can get an iterated value $P' = P^* + \Delta P$ where $\Delta P$ represents the revised value of $P$. It is given by

$$\Delta P = -Z(P^*) / \frac{\mathrm{d}Z(P^*)}{\mathrm{d}P} \tag{42}$$

in which the differentiation of $Z$ with respect to $P$ is, similar to the calculations in step 1, gained by calculating $\mathrm{d}A_{ij}/\mathrm{d}P$ which are obtained by a set of recurrence formulas of the differentiation of Eqs. (22)–(24) with respect to $P$ once $\mathrm{d}A_{10}/\mathrm{d}P$ and $\mathrm{d}B_{10}/\mathrm{d}P$ are known at the value $P = P^*$. From the expressions of boundary conditions $X = 0$ and $Y = 0$, we get

$$\frac{\mathrm{d}X}{\mathrm{d}P} = \sum_{i=1}^{\infty} \left\{ [\lambda(1-i) - 1] \frac{\mathrm{d}A_{i0}}{\mathrm{d}P} - \lambda \frac{\mathrm{d}A_{i1}}{\mathrm{d}P} \right\} = 0 \tag{43}$$

$$\frac{\mathrm{d}Y}{\mathrm{d}P} = \sum_{i=1}^{\infty} \left\{ [\mu(1-i) - 1] \frac{\mathrm{d}B_{i0}}{\mathrm{d}P} - \mu \frac{\mathrm{d}B_{i1}}{\mathrm{d}P} \right\} = 0 \tag{44}$$

These two equations are a set of linear algebraic equations with unknowns $\mathrm{d}A_{10}/\mathrm{d}P$ and $\mathrm{d}B_{10}/\mathrm{d}P$. By solving Eqs. (43) and (44), we get the roots of them. Thus, we can take the semi-analytical and semi-numerical calculations by a computer program.

*Step 3*: Replacing $P^*$ by $P'$, then, starting step 1 and step 2, we can carry out this embedded iteration method. Repeat this iteration until the condition

$$Z < 10^{-4} \tag{45}$$

is satisfied. We then obtain the numerical solution of the fundamental natural frequency for a given pressure to the pressure sensor.

With the aid of above semi-analytical and semi-numerical program, a case study for the pressure–frequency characteristic of the pressure sensor is searched for the parameters of $K = 1.0, 2.0$, $\tau = 1.7, 2.0$, and $S_0 = 0.5, 1.0, 2.0, 3.0$. In the case study, the Poisson's ratio of the plate is taken by $\nu = 0.3$. Before the numerical calculations, a numerical test taking different terms from the analytical solutions was conducted. It is exhibited that when the maximum index of $i$ in Eqs. (20) and (21) is taken to be 10, 20 and 30, the solutions converge numerically. It is found that the maximum relative difference between the numerical solutions with the maximum index of $i$ of 20 and 30 is less than 0.1%. Thus we choose the maximum index $I = 30$ in the following calculations.

Fig. 2 exhibits the pressure–natural frequency characteristic curves of dimensionless form, i.e., the $\omega^* - (qa^4/E_ph^4)$ curves. From them and Eq. (29a), there is no difficulty obtaining the pressure–natural frequency characteristic curves of the string in the pressure sensor for different parameters of initial membrane tensile force $S_0$, pre-specific gap $\tau$, and elastic constant $K$. From the results, we find that the natural frequency increases with these initial parameters, and the range of measurable pressure of the sensor is influenced by designing parameters of $\tau$ and $S_0$, but is independent of the parameter $K$.

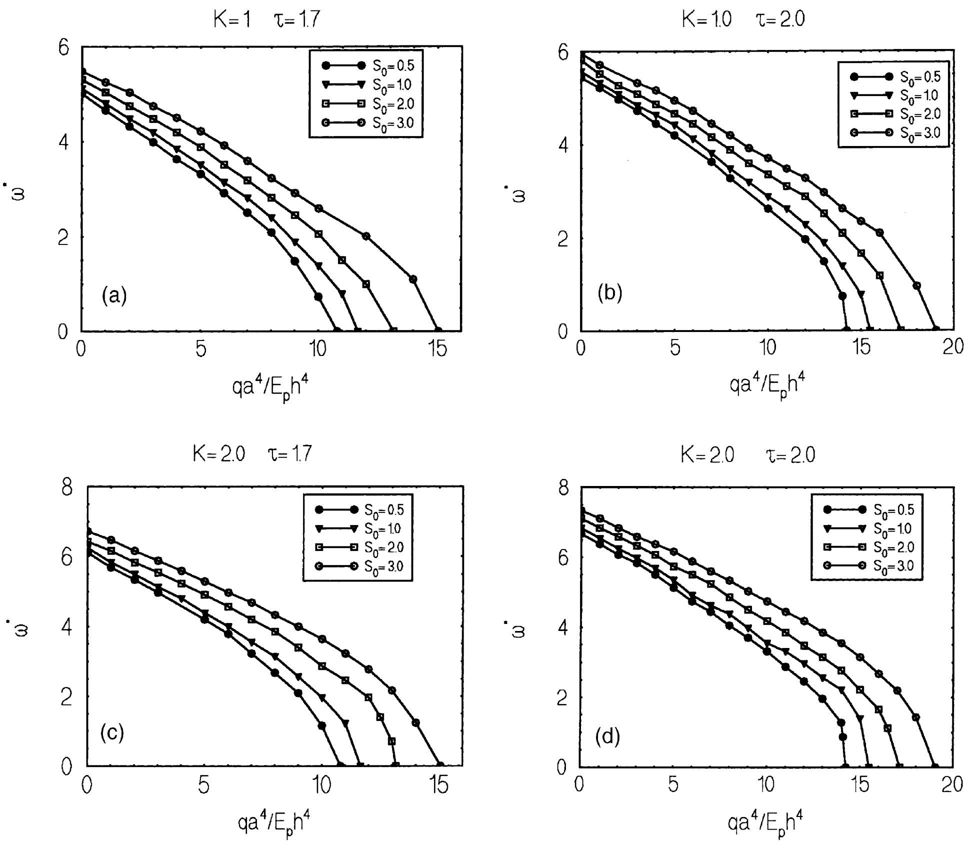


Fig. 2. Pressure–natural frequency characteristic curves of the vibrating-string type pressure sensor with different initial gap $\tau$, and initial tensile force $S_0$ ($\nu = 0.3$, $\omega_n = n\omega^* h^2/aL_0(E_p/\rho)^{1/2}$): (a) $K = 1.0$, $\tau = 1.7$, (b) $K = 1.0$, $\tau = 2.0$, (c) $K = 2.0$, $\tau = 1.7$ and (d) $K = 2.0$, $\tau = 2.0$.

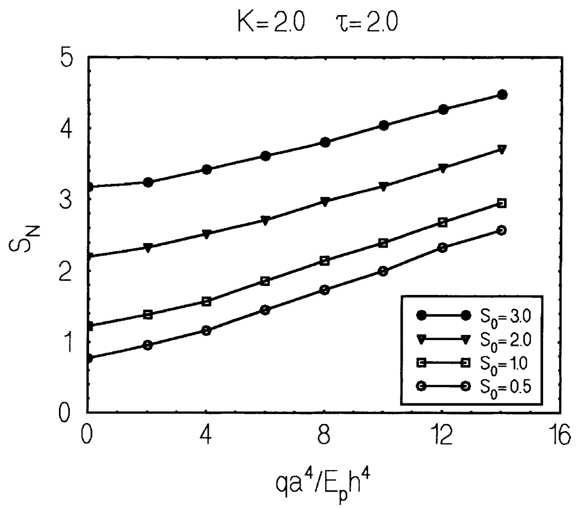


Fig. 3. Characteristic curves of radial membrane force $N_r$ at outer edge ($r = a$) varying with the applied pressure $q$ ($K = 2.0$, $\tau = 2.0$, $\nu = 0.3$, $S_N = 3(1 - \nu^2)(a^2N_r/E_ph^3)$).

In practical design of a pressure sensor, to know some mechanical quantities, e.g., internal membrane forces $N_r$ and $N_\theta$, internal bending moments $M_r$ and $M_\theta$ are useful. Here, Figs. 3 and 4 respectively display the characteristic curves of nondimensionlized radial membrane force, denoted by $S_N = 3(1 - \nu^2)(a^2N_r/E_ph^3)$, and the nondimensionlized radial bending moment, denoted by $S_M = 2[3(1 - \nu^2)]^{3/2}(a^2M_r/E_ph^4)$, at the outer edge $r = a$ varying with the applied pressure. It is found from these curves that there is small effect of the parameters of the string elasticity $k$ or $K$ and the pre-specified gap $\tau_0$ or $\tau$ on the mechanical properties (with the limit of space, here, only the curves for the case $K = 2.0$ and $\tau = 2.0$ are exhibited). From Figs. 3 and 4, one finds that the parameter of the initial membrane force $p_0$ or $S_0$ notably influences the internal membrane force $N_r$ but has almost no effect on the bending moment $M_r$.

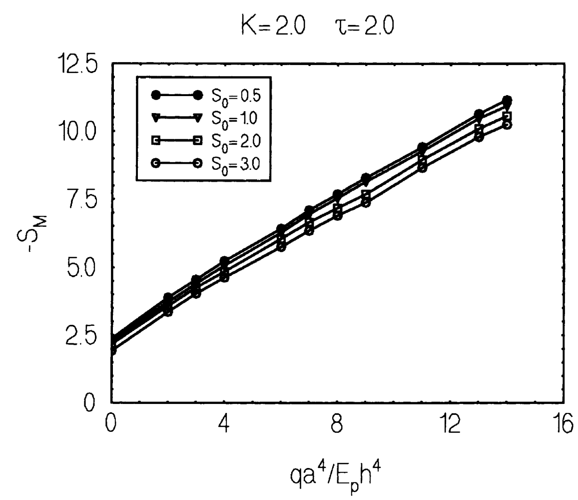


Fig. 4. Characteristic curves of radial bending moment $M_r$ at outer edge ($r = a$) varying with the applied pressure $q$ ($K = 2.0$, $\tau = 2.0$, $\nu = 0.3$, $S_M = 2[3(1 - \nu^2)]^{3/2}(a^2M_r/E_ph^4)$).

## 6. Conclusions

A semi-numerical approach for characteristic of pressure–frequency of vibration string-type pressure sensor is established by solving the von Karman's equations of plates to the pressure sensor with interaction between the plate and the string on the basis of series expansion. The numerical results of the characteristic display that the vibration frequency of the string is mainly influenced by the parameters of gap $\tau$, elongation rigidity of the string, $K$, pre-tensile membrane force of the plate, $S_0$, and the effective rigidity of the plate, i.e., a concentrated force is normally applied at the center of the plate to generate a unit deflection at the center. The larger the parameters, the higher the frequency. However, the range of measurable pressure of the sensor is dependent on the gap, the pre-tensile membrane force of the plate, and the effective rigidity of the plate, but is independent of the elongation rigidity of the string.

## Acknowledgements

The author would like to express his acknowledgement of the support related to this subject in part by a grant from the National Natural Science Foundation of China for Outstanding Young Researchers, and Science Fund of the Education Ministry of China for Excellent Young Professors in Universities.

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